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Some Recent Contributions to Panel Flutter Research

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With the objective of formulating a realistic computing program to analyze panel flutter in aerospace vehicles, plausible simplifying assumptions are examined in the light of experimental results. It is shown that in certain areas very simple analysis yields respectable results, whereas in other areas great elaboration is necessary to obtain an accurate prediction. In particular, the role played by the boundary layer flow is discussed. The attenuation and phase shift in pressure-deflection relationship caused by the boundary layer can become important under certain circumstances. Examples are given which show that the boundary layer greatly stabilizes flat plates in a transonic or low supersonic flow and circular cylindrical shells at higher Mach numbers. Some recent contributions to panel flutter research by the author and his colleagues and students at the California Institute of Technology are summarized. Although details are to be published elsewhere, a brief description of experimental results concerning flat plates and cylindrical shells is given here. The experimental and theoretical investigations taken together provide a fairly clear picture with regard to proper assumptions for an accurate analysis. Recommendations for future research in this field are given.

Nomenclature

A = $\rho U^2 L^3/MD$, ratio of dynamic pressure to panel rigidity = π^4 (Q of Ref. 1)
A_ν = coefficients of Fourier series of $z_0(x,t)$, Eq. (9)
a_m = $m = 1, 2, \ldots$, coefficients of sine series of $z_0(x,t)$, Eqs. (29) and (30)
a, a_δ = velocity of sound, in main flow and boundary layer, respectively
B_ν = coefficients of Fourier series of $z_1(x,t)$, Eq. (10)
C_ν, D_ν, E_ν = coefficients, see Eqs. (12) and (13)
D = $Eh^3/[12(1-\mu^2)]$, bending rigidity of plate
f = frequency, cps
g = structural damping factor
h = thickness of plate or shell wall
k = $\omega L/U$, reduced frequency in main flow
k_δ = $\omega L/U_\delta$, reduced frequency in boundary layer
L = chord length
M, M_δ = Mach number of main flow and of boundary layer, respectively
n = number of waves along circumference (number of nodes = $2n$)
Δp = see Eq. (33)
$p(x,t)$ = wall pressure
p, p_δ = static pressure in freestream and in boundary layer, respectively
p_m = excess of model internal pressure above p_δ, psig
p_t = wind tunnel stagnation pressure
$p_0(x,t)$ = wall pressure in potential flow without boundary layer
q = $\frac{1}{2}\rho U^2$, dynamic pressure of main flow
R = radius of middle surface of circular cylinder
r,θ,x = cylindrical polar coordinates
T, T_δ = absolute temperature in freestream and in boundary layer, respectively

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t	=	time
U	=	velocity of freestream
u,v	=	velocity components in x,y directions
V_ν	=	velocity of traveling waves, see Eq. (9)
$w(x,r,t)$, w_0	=	radial velocity on wall and amplitude, respectively
$w_{\rm rms}$	=	root mean square value of the deflection (radial or vertical) of an oscillation shell or plate
x,y	=	rectangular Cartesian coordinates; x in flow direction
z_0	=	constant
$z_0(x,y)$	=	wall displacement
$z_1(x,y)$	=	displacement of the edge of boundary layer
α_ν	=	$\nu\pi/L$, wave number, see Eqs. (9–11)
$[\alpha_1]_{mn}$, $[\alpha_2]_{mn}$	=	see Eq. (36)
β_δ	=	$[1 - M_\delta^2]^{1/2}$
$\bar{\beta}$	=	$[M^2 - 1]^{1/2}$
γ_ν	=	constants, see Eqs. (14) and (23)
δ	=	idealized boundary layer thickness
$\bar{\delta}$	=	apparent boundary layer thickness (wall to 99% freestream-velocity point)
ζ_ν	=	constants, see Eqs. (15) and (23)
κ_ν	=	$\zeta_\nu\delta$, boundary layer thickness parameter, see Eq. (18)
μ	=	Poisson's ratio
ν	=	$\pm 1, \pm 2, \ldots$, an index
ρ, ρ_δ	=	density in freestream and in boundary layer, respectively
σ_ν	=	constants, see Eqs. (19) and (23)
ϕ, ϕ_δ	=	velocity potentials in main flow and in boundary layer, respectively
ω	=	circular frequency
$\mathcal{B}_m$, $\mathcal{B}_{-m}$	=	pressure coefficients, see Eqs. (31) and (32)
$\mathfrak{M}(\kappa_\nu)$	=	ratio of wall pressure with and without boundary layer, see Eqs. (20) and (21)

1. Introduction

PANEL flutter is an oscillation of a thin-walled structure in a flow. The existence of this phenomenon has been demonstrated in laboratories, and many recondite articles have been written about the subject, but today there is neither a reliable formula to tell how to design against panel flutter nor a computing program of guaranteed accuracy. It is not a question of programming or computer size; it is a question of problem formulation.

In the field of aeroelasticity, panel flutter research has been a new experience. The physical features of the problem are simple, the oscillations are mild, but the theoretical and experimental difficulties are great. Theoretical analyses have gone far deeper into the subject than the aeroelasticians are accustomed to, yet areas of agreement between theory and experiment are limited.

Few will dispute that panel flutter falls within the realm of classical continuum mechanics. One has no difficulty writing down all the partial differential equations and boundary conditions for a panel and a flow around it, but few would tackle the full problem without heuristic simplifications. To assure numerical accuracy, one must keep the analysis from becoming too involved. Any justifications of simplifying assumptions are therefore of interest. The objective of theoretical engineering research is to find the simplest framework within which a physical phenomenon can be described satisfactorily. The mild phenomenon of panel flutter, however, turns out to be rather delicate; it cannot stand oversimplification.

About 70 papers on panel flutter published before June 1960 were reviewed by Fung.[1] By comparing various theories, it was shown there that

1) Since the differential equations are non-self-adjoint, the convergence of Galerkin or Rayleigh-Ritz method cannot be assured. In fact, the Galerkin method yields spurious flutter boundaries in the case of a membrane; but, when bending rigidity is included (so that the order of the differential equation is raised), known examples show that flutter boundaries obtained by Galerkin's method are not spurious.

2) For flat plates, the simplification of static aerodynamic force (Ackeret's formula), quasi-steady approximation, (in which the first-order terms in reduced frequency are retained), or linear piston theory (Lighthill's piston theory with linear terms retained) seems to yield good results at higher Mach numbers (say, $M > 2$). But these expressions lead to entirely erroneous flutter conditions when $M < 2^{1/2}$.

3) Panels of infinite chord length (infinite flat plates) behave quite differently from single-bay or multi-bay panels of finite chord.

4) Goldenweiser's "medium length" cylinder equation should not be used for flutter analysis because it leads to qualitatively erroneous results.

In all papers reviewed in Ref. 1, the structures are treated as elastic plates or shells according to the classical Bernoulli-Euler-Kirchhoff approximation, and the flow is considered as nonviscous potential flow. Boundary conditions usually are idealized. The motion is assumed to be of infinitesimal amplitude, so that the differential equations and the boundary conditions are linearized and that the boundary conditions are applied on the mean position of the panel rather than on the actual surface. More recent studies (since 1960) have shown that these general assumptions may need revision. In particular, the viscosity boundary layer in the flow seems to have important effects in the flutter of flat plates at lower supersonic Mach numbers and in the flutter of cylindrical shells (even at higher Mach numbers). For corrugation-stiffened panels, the classical plate theory may not apply. The boundary conditions are shown to be important. And, for cylindrical shells, our former concept about the flutter of buckled shells may need drastic revision.

It is the purpose of this article to review some work done by the present writer and his colleagues at the California Institute of Technology, with respect to a critical examination of the general assumptions mentioned in the foregoing.

2. Areas in Which Elementary Analyses Have Reasonable Success

In the formative years of aeroelasticity around 1929, pioneering investigators used quasi-static aerodynamic expressions with remarkable success. However, later advances in the speed of aircraft necessitated the development of an elaborate nonstationary airfoil theory. It was, therefore, an important event when Lighthill,[2] in 1953, pointed out that the "piston" theory furnishes a good approximation to unsteady flow at high Mach numbers. Retaining only the linear terms, the piston theory simplifies flutter analysis tremendously, as was shown first by Ashley and Zartarian.[3] Comparison of flutter boundaries computed with the linear piston theory with those using full supersonic flow theory seems to suggest that the approximation is good for M as low as 2.

The crucial test, of course, lies in comparison with experiments. Valuable early exploratory experiments by Sylvester and Baker,[4] Sylvester, Nelson, and Cunningham,[5] Jordan, Greenspon, and Goldman,[6] Eisley,[7] Kordes, Tuovila, and Guy,[8] Kordes and Noll,[9] Tuovila and Presnell,[10] Dixon, Griffith, and Bohon,[11] and many unpublished experiments by Boeing, Martin (Denver), and other companies have thrown much light on the flutter problem, but generally these experiments are too broad or too complicated to permit a detailed comparison with theory.

Since in panel flutter many factors are important, including the flow conditions, boundary layer, the edge restraint conditions, the initial curvature, and the tension or compression loads in the plate due to mechanical or thermal effects, a comparison between theory and experiment can be done only if both the theory and the experiment cover the same ground. In other words, all experimental conditions must be analyzed

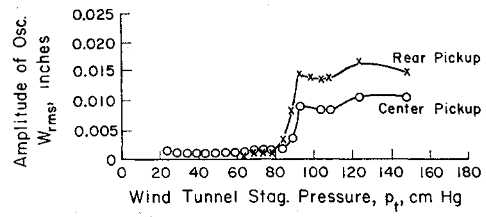

Fig. 1 Flutter amplitudes for 0.015-in. flat panels at M = 2.81; chord length, 9.25 in. (Jet Propulsion Laboratory tests, Anderson[12])

and corrected in the theory before the comparison. Such an evaluation obviously is important and is not easy.

A series of experiments by Anderson[12] lends support to the claim that, for the flutter of flat panels, the potential flow linear piston theory yields reasonable agreement with experimental results at higher Mach numbers; this will be described briefly here. Anderson's experiments were performed at California Institute of Technology in the Jet Propulsion Laboratory 12-in. supersonic wind tunnel at Mach number 2.81. His models were mounted in a wedge-shaped aerodynamic frame similar to that which was used previously by Eisley[7] for buckled panels in the same tunnel. A number of flat, rectangular panels were designed to study two-dimensional flutter. These panels were clamped at front and rear with free sides that extended into the boundary layer at the sides of the tunnel. A flexure arrangement was used at the rear edge to minimize midplane stresses. These panels fluttered in a two-dimensional mode. The experimental results, stated in terms of a parameter $A = \rho a U L^3/D$, are about 35% higher than those predicted by Houbolt[13] on the basis of linear piston theory. Stated in terms of plate-thickness chord-length ratio required to prevent the two-dimensional flutter of a flat, unstressed plate, the experimental results are about 15% higher than the theoretical values.

Some details are of interest. The definition of flutter will be discussed first. In a linear theory, the critical flutter condition represents a sustained harmonic oscillation. On the unstable side of the flutter boundary, the amplitude of oscillation grows exponentially with increasing time; on the stable side, it subsides exponentially. Experimentally, an entirely different phenomenon is observed. The panel lies in a flow that is more or less turbulent, and so on the stable side one expects a random oscillation of small amplitude. On the unstable side, the growth of amplitude is limited by the nonlinear effects of membrane tension, and so one expects a steady limit cycle oscillation. A typical amplitude of oscillations vs tunnel stagnation pressure at Mach number 2.81 for a flat plate of thickness 0.015 in., chord length 9.25 in., at zero midplane stress, is shown in Fig. 1. The point where a rather sudden increase in amplitude of oscillation occurs is regarded as the critical flutter condition.

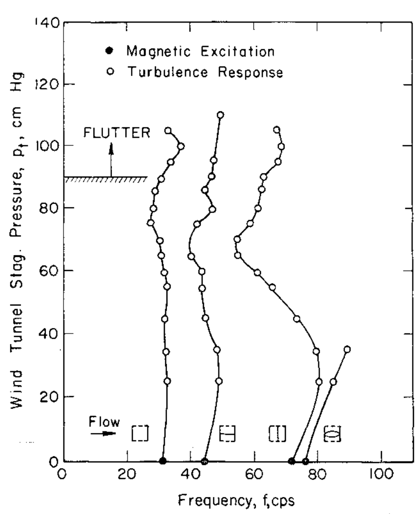

Fig. 2 Resonance frequencies for a flat panel 0.0153-in. thick; two-dimensional flutter; M = 2.81

The frequency response also is of interest. In a linear theory, the time dependence of the solution is assumed to be of the form $e^{i\omega t}$. In a vacuum, a panel in free vibration has a real-valued frequency spectrum, $\omega_1 \leqslant \omega_2 \leqslant \omega_3 \leqslant \ldots$. If aerodynamic damping is ignored, the theoretical frequency spectrum changes continuously with the total pressure of the flow (other parameters being fixed). A critical total pressure $p_{t_{cr}}$ is reached when two frequencies coalesce. At $p_t > p_{t_{cr}}$, the pair of coalesced frequencies becomes a pair of complex conjugate numbers, one of which means a divergent oscillation. If aerodynamic damping is not ignored, the situation is similar, except that before the frequency coalescence the time factors ω_n are complex with positive imaginary part, so that the motion is convergent; this situation continues until a value of p_t is reached at which a pair of ω_n's coalesce; then at another, higher value of p_t, say $p_{t_{cr}}$, one of the roots ω_n will have a negative imaginary part, corresponding to a divergent oscillation. This theoretical feature has come to be accepted widely, and many engineers have based their design concepts on the frequency spectrum: the closeness or coincidence of the frequencies of two vibration modes in a vacuum is regarded as a criterion of panel flutter. This concept, of course, is unfounded, although it does have heuristic value.

In wind tunnel experiments, free vibration conditions do not prevail. Anderson's frequency surveys during the approach to flutter failed to turn up any frequency coalescence. Figure 2 shows a typical set of curves of resonant frequencies vs stagnation pressure of the flow. The resonant frequencies in the preflutter region were followed either by using a magnet excitation or by a harmonic analysis of the random oscillations of the panel. In the case shown in Fig. 2, it was possible to follow these frequencies through the onset of flutter. It was easy to see that flutter occurred at one of the resonant frequencies. The amplitude of the response at this frequency grew rapidly as flutter started, completely dwarfing the response at the other frequencies.

These examples serve to show the difference between the problem that is commonly formulated in theory and that which is observed conveniently in experiments. The comparison of theory and experiment, is, therefore, intuitive in character, and agreement or disagreement cannot be accepted on face value. For instance, from the absence of frequency coalescence in Anderson's experiments, it does not follow immediately that the linear theory is all wrong, because although the theory examines free vibration, the experiment observes a stochastic forced oscillation due to wind tunnel turbulences. Thus, at p_t greater than $p_{t_{cr}}$, although one theoretical mode whose frequency ω_n becomes complex with negative imaginary part passes into a limit cycle, other modes still can be excited under a random loading. To justify fully a comparison, the nonlinear aspects must be examined.

A large number of factors are ignored in comparing Houbolt's theory and Anderson's experiments, such as the boundary layer, the initial imperfections of the plate, the uncertainties of the clamped edge conditions, the small static pressure differential across the plate between a cavity below the plate and the freestream above, and the acoustic action of the cavity. The influences of these factors, except that of the boundary layer, are discussed in some details in Ref. 14. Application of these corrections will not alter the comparison very much. It may be concluded that the prediction of critical thickness ratio based on the linear potential theory agrees with experiments to within approximately 15%. One must regard the linear potential theory as remarkably successful in the problem of flutter of flat plates at Mach number 2.8.

If the nonlinear problem is studied in the future, it would be interesting to examine and extend also Anderson's tests on slightly curved plates, which also are reported in Ref. 12. The fact that slightly curved plates (with cylindrical generators perpendiular to the flow) remained almost motionless up to the flutter boundary, where large amplitude flutter occurred (much more violent than the corresponding flat plate case), must be a revelation of the difference in nonlinear effects in flat and curved plates.

3. Difficulties with Flat Plates at Lower Supersonic Mach Numbers

It is expected that the linear piston theory becomes inapplicable at lower supersonic Mach numbers, but it is unexpected and disappointing that the classical linearized supersonic flow theory of an inviscid fluid also fails to predict the flutter condition correctly.

A series of experiments was performed by Lock[14] in the Guggenheim Aeronautical Laboratory, California Institute of Technology 4 × 10 in. transonic wind tunnel at Mach numbers between 0.85 and 1.5. The model installation was designed to represent two-dimensional conditions. In these experiments, the wind tunnel stagnation pressure was fixed, but the Mach number could be varied continuously. A set of plates of different thicknesses was used. The experimental definition of flutter and the frequency response measurements were similar to those discussed in the preceding section. Figure 3 shows the flutter boundary, where h/L represents the panel thickness to chord length ratio. The plates were clamped at the front, free on the sides that were parallel to the flow and next to the tunnel wall, and attached at the rear edge to a flexure that could translate to relieve membrane stress in the plate.

Theoretical results[14] for these plates based on the linear, inviscid supersonic aerodynamics also are shown in Fig. 3. (Curve relating to boundary layers will be discussed later.) The effects of flexure, the initial curvature, damping, membrane stress, static pressure differential across the plate, and the number of modes used in the calculations, etc., were examined; these effects will not change the theoretical curves substantially.[14] After these detailed considerations, it was concluded that the effect of the viscosity boundary layer might be the main cause of the discrepancy between the theoretical and experimental flutter boundaries at Mach numbers below 1.4.

The suspicion about the influence of boundary layer was strengthened later when a series of experiments was performed on cylindrical shells at the NASA Ames Research Center's unitary wind tunnel in June 1961. In that test, the flow was parallel to the cylinder axis, and the model was designed so that it should flutter according to any one of the seven theories proposed at that time (Miles,[15] Hedgepeth and Leonard,[16] Stepanov,[17] Voss,[18] Strack and Holt,[19] Shulman,[20] Krumhaar[21]). The instrumentation employed should be able to pick up any one of the modes of flutter proposed in these theories. However, nothing happened; the shell was stable throughout the operating range of the tunnel, as long as the critical buckling condition was not approached (at which none of the theories applies). Since the forementioned theories cover a wide variety of physical assumptions and mathematical techniques of solution, it was felt that the fault lies in an assumption common to them all: that the fluid is considered inviscid.

4. Experiments on Circular Cylindrical Shells

It long has been suspected that all published theories of flutter of circular cylindrical shells[15–21] yield pessimistic results about shell instability, because many missiles and spacecraft that had flown successfully had skin thicknesses thinner

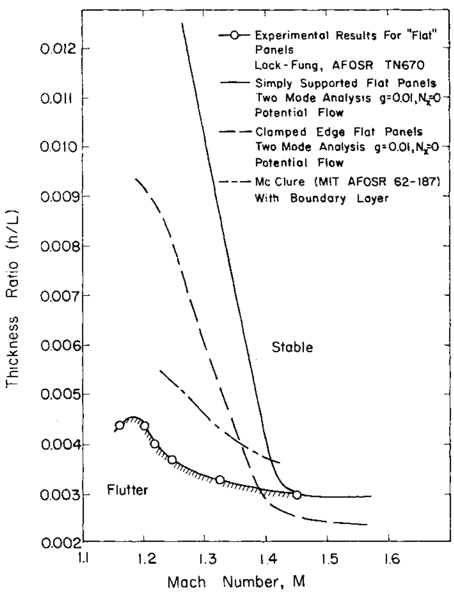

Fig. 3 Comparison of the theoretical flutter boundaries with experimental results

than theoretically recommended gages for safety against flutter. The June 1961 tests confirmed this suspicion. Following that first, negative experiment, thinner shells were made, and the tests were repeated at the NASA Ames 8 × 7 ft tunnel in May 1962. Flutter then was observed, a very brief description of which will be given below (see Ref. 22 for details).

The two models used in the Ames experiments were thin-walled monocoque circular cylinders of electroplated copper with a radius of 8 in. and length of 16 in. The wall thickness of these models were 0.0032 and 0.0040 in.; the radius to thickness ratios R/h were, therefore, 2500 and 2000, respectively. The flow was parallel to the cylinder axis. The models were made by an electroplating process. In this process, the relatively heavy end rings first were fastened to a mandrel, on which a layer of wax of about 1-in. thick was applied. The waxed mandrel then was machined carefully to the right dimensions, sprayed with conductive silver paint, and electroplated. When the wax was removed after plating, a seamless cylinder welded integrally to the end rings was obtained.

The thin-shell model then was mounted on a heavy concentric cylinder ("center body") so that the outside of the shell could be exposed to a flow while the inside was exposed to a chamber that could be pressurized. A forebody that assured uniform flow over the model and an afterbody that connected the model to the sting were added. The model dimensions are shown in Fig. 4. A photograph of the model in the wind tunnel is shown in Fig. 5. The entire model support system was a duct, not unlike that of a jet engine, to minimize the frontal area and to shorten the forebody.

For instrumentation, three inductance pickups were used. These pickups measured the deflection of the shell without touching it. One pickup was fixed, and two were mounted on a gear-driven drum that could rotate concentrically with the centerbody. One of these last-named pickups was fixed on the drum, whereas the other could translate parallel to the cylinder axis (see Fig. 6). The continuous motions of these pickups made it possible to record the longitudinal and circumferential displacements continuously. These displacements were recorded on magnetic tape with an FM system. Spectral and phase angle relationship analyses were made from these records. In this way it was possible to discriminate various flutter modes: short wave length traveling waves (Miles), long traveling waves (Hedgepeth and Leonard), symmetric flutter (Krumhaar, Stepanov, and

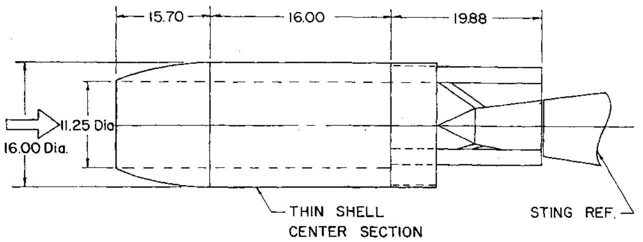

Fig. 4 Sketch of cylinder model used in Ames tests; dimensions in inches

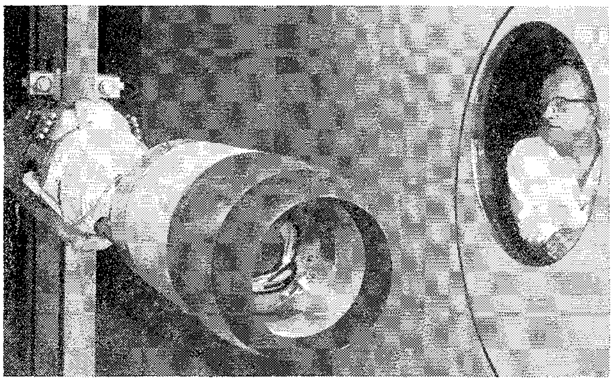

Fig. 5 Photograph of model in the Ames Unitary Plan wind tunnel

Voss), and flutter with many circumferential nodes (Hedgepeth and Leonard, Voss, Stepanov, Holt and Strack, etc.).

The remarkably versatile Ames Unitary Plan wind tunnel provided mechanisms for continuous variation of Mach number and stagnation pressure. Searching of flutter could be done, therefore, by varying M (in the range 2.5 to 3.5), p_t (in the range of 4 to 50 psia) and p_m, defined as the difference between the model internal pressure and the static pressure in the flow (in the range +4 psig, when hoop stress in the shell is in tension, to −0.02 or −0.04 psig, when the shells buckled).

Extensive still-air vibration tests were performed by Watts.[23] The vibration characteristics were similar to what was found before by Fung, Sechler, and Kaplan.[24] Theoretical background of the method of acoustic excitation used in Ref. 24 was examined by Keith and Krumhaar.[28] Interpretation of vibration results due to acoustic excitation may become complicated. To avoid confusion, an electrodynamic excitor was used. The damping coefficient, expressed in terms of the "structural damping factor g," was estimated to be 0.0025. A brief sketch of the results follows.

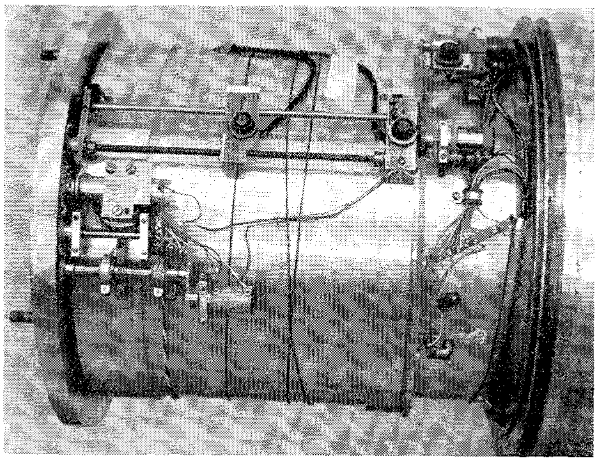

Fig. 6 Photograph of model instrumentation: pickup traversing mechanism

A. Motion

When the wind tunnel was started at p_t = 4 psia and p_m = 4 psig, rough vibrations of the shell occurred as long as the flow was subsonic, but the shell became very quiet as soon as supersonic flow was established (root mean square amplitude of oscillation of order 0.0001 in. or about $\frac{1}{25}$ thickness). The small motion in this regime is random in amplitude, as is shown in the lower oscillogram traces in Fig. 7 (part c).

As the stagnation pressure and model internal pressure were varied, two other types of motion occurred under certain conditions. The middle traces (part b) in Fig. 7 illustrate a clean periodic oscillation. The upper traces (part a) in Fig. 7 indicate a kind of beating between two or more sinusoidal oscillations of slightly different frequencies. The corresponding energy spectra obtained by harmonic analyses are shown in Fig. 8, which corroborates the foregoing observation. Types a and b usually were associated with fairly large amplitudes of oscillations (of order 2 or 3 skin thicknesses), only these types qualify for the term flutter.

B. Flutter Condition

As in the case of flat plates, an experimental definition of flutter is necessary, because the flutter condition formulated

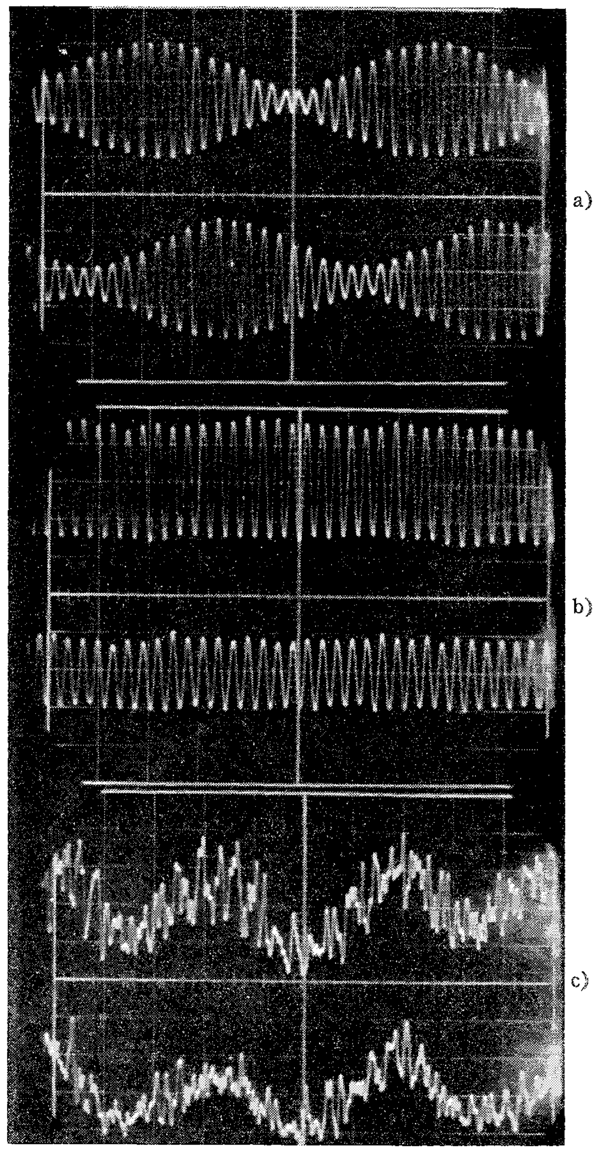

Fig. 7 Types of cylinder wall motion: a,b) shell response during flutter; c) shell response prior to flutter. Estimated sweep speed was of the order of 0.005 sec per major division of the scale. Vertical scales for a and b are much larger than for c

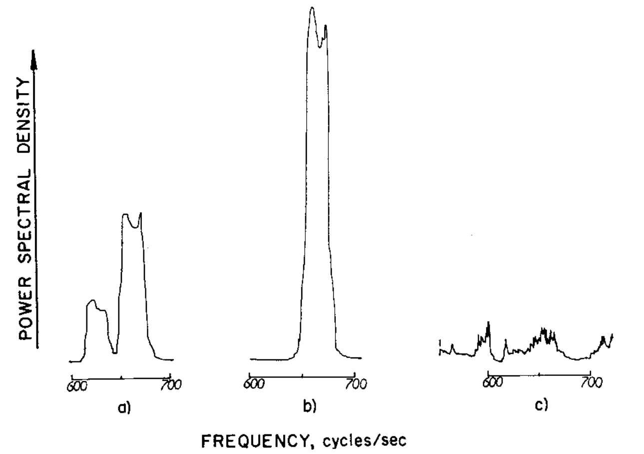

Fig. 8 Power spectra corresponding to oscillations depicted in Fig. 7

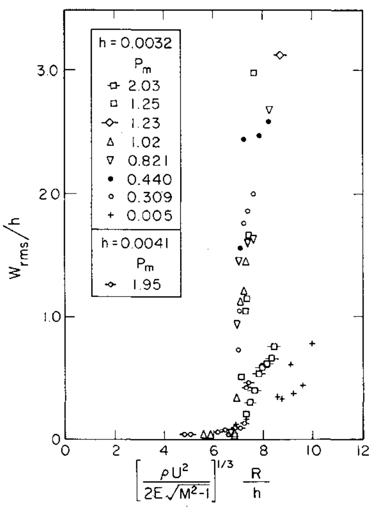

Fig. 9 Amplitude of flutter as a function of the parameter $\frac{R}{h}\left[\frac{q}{E(M^2-1)^{1/2}}\right]^{1/2}$

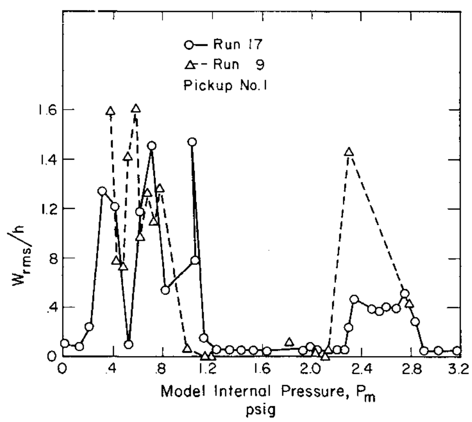

Fig. 10 Amplitude of flutter as a function of model internal pressure

in linearized theories (an exponential increase of amplitude with increasing time) is unobservable. In practice, only the limiting cycle oscillations can be seen. Therefore, one is faced with the problem of relating the amplitude of a nonlinear oscillation with the linear concept of instability. Intuitively, a plausible relationship can be established, as just indicated in the flat plate case, but a rigorous treatment is wanting.

For cylindrical shells, a plot of the root mean square values of the amplitude of the shell displacement vs the stagnation pressure of the flow yields a set of data as shown in Fig. 9. A rather sudden rise of amplitude occurred as p_t exceeds certain critical value. This critical value, as absorbed in the parameter $(R/h)(q/E\bar{\beta})^{1/3}$, seems to yield the experimental critical condition:

$$(q/\bar{\beta}E)^{1/3}(R/h) \doteq 7$$

which seems to be independent of the internal pressure p_m when it is positive. Let this be called the critical flutter condition. Figure 9 shows that beyond the critical flutter condition the rate of rise of the amplitude of oscillations depends on the internal pressure in a rather complicated manner. To emphasize the last point, in Fig. 10 a plot is presented of the rms amplitude of radial displacement at pickup number 1 at $M = 2.49$ and $p_t = 4.98$ psi. The details of this curve are hard to understand. It is worthy to note that, as the model internal pressure tended to zero, the amplitude suddenly became very small. At negative pressure the amplitude increased very rapidly as the negative pressure was increased, but when buckling was approached under lateral hydrostatic pressure, the amplitude again decreased. At large buckles the shell was very stable again.

C. Flutter Mode

An analysis of the correlation of the displacement histories and the phase relationship of the oscillations at different pickups, when these pickups were moved continuously both in the axial and in the circumferential directions, gives the flutter mode. In the Ames tests the instrumentation was sufficient to discriminate among various types of traveling and standing waves but was not, because of the large distances traversed, accurate enough to give the mode shapes quantitatively. An example will be shown here, for a flutter point at $M = 2.49$, $p_t = 4.85$ psi, $p_m = 1.23$ psi, of the 0.0032-in.-thick shell, the variations of the rms amplitude of the oscillation in the axial and circumferential directions are shown in Fig. 11. The corresponding phase relationship of the motion sensed by the moving pickups relative to that sensed by the fixed pickup revealed a phase shift of about 20° in axial traverse. On circumferential traverse, the phase angle oscillates between 0° and 180° seven times over a semicircle. The power spectrum of the motion at the fixed pickup is given in Fig. 12. From Figs. 11 and 12, one may conclude that the flutter mode at this point consists of many circumferential waves, i.e., having many nodal lines parallel to the cylinder axis ($n = 14$ waves in this example), whereas in the longitudinal direction there was no nodal line between the ends.

Analyses of other flutter points lead to similar results. It must be concluded from these measurements that traveling waves of small wavelengths as assumed by Miles and the circular symmetric modes as considered by Krumhaar, Stepanov, etc. were not observed.

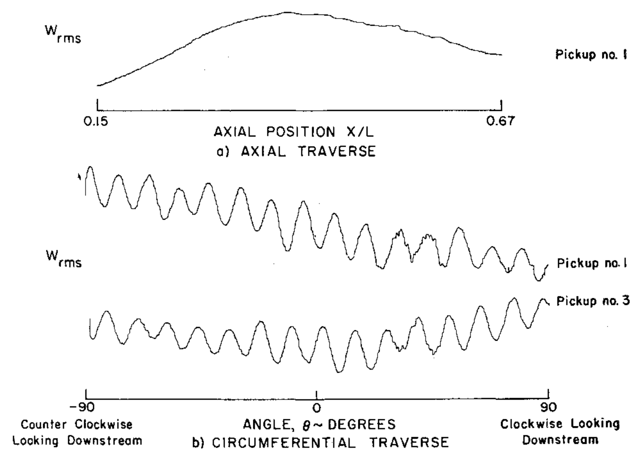

Fig. 11 Flutter mode determination: longitudinal and circumferential traverses

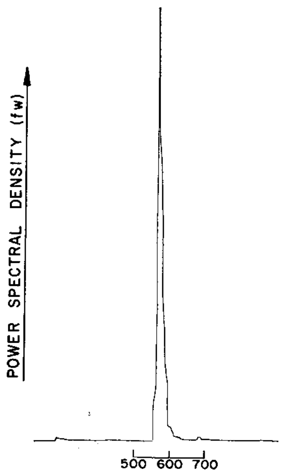

Fig. 12 Power spectrum of 0.0032-in. shell in flutter mode depicted in Fig. 11. The spectrum was obtained using a 2-cps bandwidth filter

D. Buckling

At a negative internal pressure $p_m = -0.035$ psi, the hydrostatic pressure was sufficiently large to cause buckling of the 0.0040-in. shell. The amplitude of buckles increases rapidly as the hydrostatic pressure is increased beyond the critical buckling pressure. The buckling mode of the 0.0040-in.-thick shell, measured by the same instrumentation, consists of 15 circumferential waves with nodal lines parallel to the cylinder axis but no circumferential nodal lines between the ends. As mentioned previously, it is interesting to note that the amplitude of flutter became very large as the buckling condition was approached. The largest amplitude of oscillation occurred at a negative internal pressure of approximately $\frac{1}{3}$ of the critical buckling pressure. As the shell buckled, the amplitude of oscillation became small again; the cylinder became quite stable when the buckles deepened. One perhaps may think of the shell with large buckles as a corrugated shell, the critical flutter speed of which is high. With this point of view, one should expect an entirely different aeroelastic behavior of a cylinder with respect to buckling due to axial compression.

5. Role of Boundary Layer Flow

The boundary layer flow over a solid body has important influence in many phenomena in aeroelasticity. In buffeting or stall flutter, the boundary layer may detach from the solid wall, causing complete change in flow pattern. In panel flutter, the boundary layer causes changes in amplitude and phase relationship between pressure and wall displacement.

To formulate a simple idealized problem, which sheds some light on the role of boundary layer in panel flutter, consider the following: an infinite flat plate oscillates harmonically in a standing sine wave with straight nodal lines perpendicular to the flow. The unperturbed flow in the half-space above the plate is a uniform supersonic flow of Mach number M; in between the supersonic flow and the plate is a layer of parallel uniform subsonic flow of constant thickness δ and Mach number M_δ. The interface between the supersonic flow and the subsonic layer is a vortex sheet of constant strength. Assume that the amplitude of oscillation of the plate is small compared with the thickness of the subsonic layer δ and that the perturbed flows in both the subsonic layer and the supersonic half-space are isentropic and irrotational. The problem is to relate the pressure distribution on the plate with the surface displacement.

Although the idealization just named is so severe that the conditions are unrealizable, it does preserve two features that are important to the problem. First, the entire flow outside of the boundary layer is influenced by the oscillation of the wall; second, the pressure across the boundary layer does not remain constant but is variable throughout the thickness.

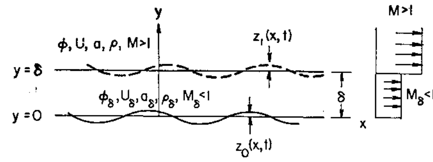

Fig. 13 Notations for an idealized boundary layer

Let (u,v) be the perturbation velocity and ϕ be the perturbation velocity potential, so that $\nabla\phi = (u, v)$ and $(u^2 + v^2)/U^2 \ll 1$. The uniform main flow velocity, speed of sound, fluid density, and Mach number will be denoted by U, a, ρ, M, respectively, and the corresponding quantities in the subsonic layer will be indicated with a subscript δ (see Fig. 13). The linearized equations of motion are (see Ref. 26, pp. 419, 432)

$$\frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2} + \frac{2M}{a}\frac{\partial^2\phi}{\partial x\partial t} + \bar{\beta}^2\frac{\partial^2\phi}{\partial x^2} = \frac{\partial^2\phi}{\partial y^2} \qquad (y \geq \delta) \quad (1)$$

$$\frac{1}{a_\delta^2}\frac{\partial^2\phi_\delta}{\partial t^2} + \frac{2M_\delta}{a_\delta}\frac{\partial^2\phi_\delta}{\partial x\partial t} - \beta_\delta^2\frac{\partial^2\phi_\delta}{\partial x^2} = \frac{\partial^2\phi_\delta}{\partial y^2} \qquad (2)$$

for $(0 \leq y \leq \delta)$

$$\bar{\beta}^2 = M^2 - 1 > 0 \qquad \beta_\delta^2 = 1 - M_\delta^2 > 0$$

The boundary conditions are $(-\infty < x < \infty)$, $(-\infty < t < \infty)$

$$y = 0: \quad \frac{\partial\phi_\delta}{\partial y} = \frac{\partial z_0}{\partial t} + U_\delta\frac{\partial z_0}{\partial x} \qquad (3)$$

$$y = \delta - 0: \quad \frac{\partial\phi_\delta}{\partial y} = \frac{\partial z_1}{\partial t} + U_\delta\frac{\partial z_1}{\partial x} \qquad (4)$$

$$y = \delta + 0: \quad \frac{\partial\phi}{\partial y} = \frac{\partial z_1}{\partial t} + U\frac{\partial z_1}{\partial x} \qquad (5)$$

$$y = \delta: \quad p(x,\delta^{-0},t) = p(x,\delta^{+0},t) \qquad (6)$$

i.e.,

$$-\rho_\delta\left(\frac{\partial\phi_\delta}{\partial t} + U_\delta\frac{\partial\phi_\delta}{\partial x}\right) = -\rho\left(\frac{\partial\phi}{\partial t} + U\frac{\partial\phi}{\partial x}\right) \text{ at } y = \delta$$

$$y \to \infty: \text{ finiteness and radiation condition} \qquad (7)$$

Consider a standing wave on the wall:

$$z_0(x, t) = e^{i\omega t}\sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{L} \qquad (8)$$

In the process it will be shown that the perturbed oscillation of the boundary layer [interface $z_1(x, t)$] consists of not only a standing wave but also a traveling wave. The traveling wave becomes more important to the stability of the panel as the supersonic Mach number is reduced ($M \to 1$). Thus a surprising reconciliation between the "standing wave theory of panel flutter" and the "traveling wave theory of panel flutter" (Miles[27]) is obtained. Write

$$z_0(x, t) = \sum_{\nu=-\infty}^{\infty} A_\nu e^{i(\omega t + \alpha_\nu x)} = \sum_{\nu=-\infty}^{\infty} A_\nu e^{i\alpha_\nu(x - V_\nu t)} \qquad (9)$$

$$z_1(x, t) = \sum_{\nu=-\infty}^{\infty} B_\nu e^{i(\omega t + \alpha_\nu x)} = \sum_{\nu=-\infty}^{\infty} B_\nu e^{i\alpha_\nu(x - V_\nu t)} \qquad (10)$$

where

$$\alpha_\nu = \nu\pi/L \qquad V_\nu = -\omega/\alpha_\nu \qquad (11)$$

It is easy to see that the differential Eqs. (1) and (2) have the solutions

$$\phi = \sum_{\nu=-\infty}^{\infty} E_\nu e^{i(\omega t + \alpha_\nu x - \gamma_\nu y')} \qquad (y' = y - \delta) \quad (12)$$

$$\phi_\delta = \sum_{\nu=-\infty}^{\infty} [C_\nu \sin\zeta_\nu y + D_\nu \cos\zeta_\nu y] e^{i(\omega t + \alpha_\nu x)} \quad (13)$$

where

$$\gamma_\nu^2 = \frac{\omega^2}{a^2} + \frac{2M\omega\alpha_\nu}{a} + (M^2 - 1)\alpha_\nu^2$$

$$\gamma_\nu = \frac{\pi}{L}\left[M^2\left(\frac{k}{\pi} + \nu\right)^2 - \nu^2\right]^{1/2} (\nu = \pm 1, \pm 2, \ldots) \quad (14)$$

$$\zeta_\nu^2 = \frac{\omega^2}{a^2_\delta} + \frac{2M_\delta\omega\alpha_\nu}{a_\delta} + (M_\delta^2 - 1)\alpha_\nu^2$$

$$\zeta_\nu = \frac{\pi}{L}\left[M_\delta^2\left(\frac{k_\delta}{\pi} + \nu\right)^2 - \nu^2\right]^{1/2} \quad (\nu = \pm 1, \pm 2, \ldots) \quad (15)$$

and

$$k = \omega L/U \qquad k_\delta = \omega L/U_\delta \quad (16)$$

are reduced frequencies.

As ν ranges over $-\infty$ to ∞, γ_ν^2, ζ_ν^2 can have both plus and minus signs. For γ_ν, the selection of branches of the multivalued function must be based on the radiation and finiteness conditions at ∞. For ζ_ν, it is arbitrary. Choose γ_ν, ζ_ν to be real and positive if γ_ν^2, ζ_ν^2 were real and positive, and γ_ν, ζ_ν to be imaginary with complex argument $-\pi/2$ (on the lower half-plane) if γ_ν^2, ζ_ν^2 were real and negative. With this choice, $\phi(x, y; t)$ represents an outgoing wave in the y direction if $\gamma_\nu^2 > 0$ or decreases exponentially as $y \to \infty$ if $\gamma_\nu^2 < 0$. Note that, if ω were allowed to be complex, then a divergent wall oscillation, $Im\ \omega < 0$, will correspond to a complex-valued γ_ν with $Im\ \gamma_\nu < 0$. Then ϕ decreases with increasing y, as the radiation condition would imply.

The constants E_ν, C_ν, D_ν can be determined from the boundary conditions (3–5). Then condition (6) gives the desired result:

$$B_\nu/A_\nu = 1/(\cos\kappa_\nu - \sigma_\nu \sin\kappa_\nu) \quad (17)$$

where

$$\kappa_\nu = \zeta_\nu\delta \quad (18)$$

$$\sigma_\nu = -i\frac{\rho}{\rho_\delta}\frac{(\omega + U\alpha_\nu)^2}{(\omega + U_\delta\alpha_\nu)^2}\frac{\zeta_\nu}{\gamma_\nu} = -i\frac{\rho U^2}{\rho_\delta U_\delta^2}\left(\frac{(k/\pi) + \nu}{(k_\delta/\pi) + \nu}\right)^2\frac{\zeta_\nu}{\gamma_\nu} \quad (19)$$

Thus the problem is solved. The pressure on the wall is obtained:

$$p(x,0;t) = \rho\sum_{\nu=-\infty}^{\infty} i(\omega + U\alpha_\nu)^2\frac{A_\nu}{\gamma_\nu}\mathfrak{M}(\kappa_\nu)e^{i(\omega t + \alpha_\nu x)} \quad (20)$$

where

$$\mathfrak{M}(\kappa_\nu) = \frac{\cos\kappa_\nu + (1/\sigma_\nu)\sin\kappa_\nu}{\cos\kappa_\nu - \sigma_\nu\sin\kappa_\nu} \quad (21)$$

The function $\mathfrak{M}(\kappa_\nu)$ represents the influence of the boundary layer. Since $\mathfrak{M}(\kappa_\nu) \to 1$ as $\delta \to 0$, the expected solution of an oscillating wall in a uniform supersonic main flow is obtained:

$$p_0 = \text{wall pressure in potential flow}$$
$$= \rho\sum_{\nu=-\infty}^{\infty} i(\omega + U\alpha_\nu)^2\frac{A_\nu}{\gamma_\nu}e^{i(\omega t + \alpha_\nu x)} \quad (22)$$

To examine the nature of the solution, it is sufficient to consider $\nu = 1$ and -1, corresponding to traveling waves of wave length $2L$ up- and downstream, respectively; other values of ν merely represent waves of shorter wave length $2L/|\nu|$. As traveling waves, the individual terms can be understood better in terms of the phase velocity V_ν relative to the panel [see Eqs. (9–11)]. Thus

$$\gamma_\nu = (|\alpha_\nu|/a)[(U - V_\nu)^2 - a^2]^{1/2}$$
$$\zeta_\nu = (|\alpha_\nu|/a_\delta)[(U_\delta - V_\nu)^2 - a_\delta^2]^{1/2} \quad (23)$$
$$\sigma_\nu = -i\frac{\rho U^2}{\rho_\delta U_\delta^2}\left(\frac{U - V_\nu}{U_\delta - V_\nu}\right)^2\frac{\zeta_\nu}{\gamma_\nu}$$
$$p(x,0;t) = \rho\sum_\nu i\alpha_\nu(U - V_\nu)^2\frac{A_\nu}{\gamma_\nu}\mathfrak{M}(\kappa_\nu)e^{i\alpha_\nu(x - V_\nu t)}$$

It is seen that γ_ν, ζ_ν vanish when, respectively,

$$U - V_\nu = \pm a \qquad U_\delta - V_\nu = \pm a_\delta \quad (24)$$

Further, σ_ν becomes either zero or indeterminate if

$$U = V_\nu \qquad U_\delta = V_\nu \quad (25)$$

These correspond to steady transonic flows or static conditions, respectively, with respect to an observer moving with the traveling wave. In the former case, Eq. (24), the approximation is not valid.

The influence of boundary layer is exhibited by the function $\mathfrak{M}(\kappa_\nu)$, which is the ratio of pressures in flows with and

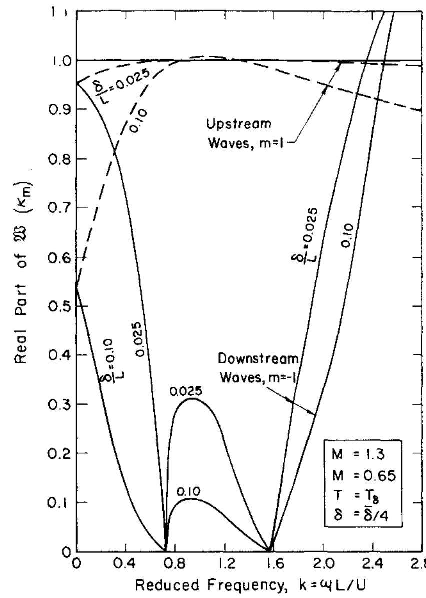

Fig. 14a Real part of the function $\mathfrak{M}(\kappa_\nu)$, ratio of aerodynamic pressure on wall with and without boundary layer; traveling waves; $M = 1.3$, $M_\delta = 0.65$

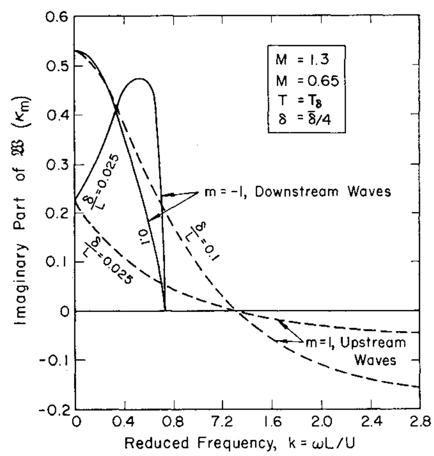

Fig. 14b Imaginary part of the function $\mathfrak{M}(\kappa_\nu)$, ratio of aerodynamic pressure on wall with and without boundary layer; traveling waves; $M = 1.3$, $M_\delta = 0.65$

without boundary layer. If one sets $T = T_\delta$, so that $p = p_\delta$, $\rho = \rho_\delta$, $a = a_\delta$ for the undisturbed flow, then $\mathfrak{M}(\kappa_\delta)$ depends on the parameters M, M_δ, δ/L, and k.

The behavior of $\mathfrak{M}(\kappa\nu)$ is exhibited numerically in Fig. 14, which is very instructive in showing that a wide variation in $\mathfrak{M}$ is possible. On the other hand, a standing wave of the wall,

$$z_0(x,t) = z_0\, e^{i\omega t} \sin(\pi x/L) \tag{26}$$

corresponds to

$$A_{\pm 1} = \pm z_0/2i \qquad \alpha_{\pm 1} = \pm \pi/L \tag{27}$$

$$z_1(x,t) = \frac{1}{\cos\kappa_1 - \sigma_1 \sin\kappa_1}\, z_0 e^{i\omega t} \sin\frac{\pi x}{L} - \left(\frac{1}{\cos\kappa_{-1} - \sigma_{-1}\sin\kappa_{-1}} - \frac{1}{\cos\kappa_1 - \sigma_1 \sin\kappa_1}\right)\frac{z_0}{2i}\, e^{i[\omega t - (\pi x/L)]} \tag{28}$$

This shows clearly that the edge of the boundary layer appears as an attenuated wall oscillation plus a traveling wave.

The foregoing results cannot be used directly in analyzing panel flutter of finite panels, because the influence of a leading edge is not clarified. It is known that for $1 < M < 1.4$ the potential theory of supersonic flow shows a strong leading edge effect (no disturbance in front of the leading edge). For the idealized boundary layer (uniform subsonic flow in $0 < y < \delta$), the writer has worked out a complete solution for an arbitrary oscillation of a finite wall, but the results are complicated. For a qualitative examination, two simple alternatives are suggested. The first ignores the leading edge effect, treating a finite panel as one period of an infinite wall. Thus, if

$$\begin{aligned} z_0(x,t) &= 0 && \text{for } x < 0 \\ &= e^{i\omega t} \sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{L} && \text{for } x \geqslant 0 \end{aligned} \tag{29}$$

it is assumed that the wall pressure $p(x,t)$ is the same as that induced by

$$z_0(x,t) = e^{i\omega t} \sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{L} \quad \text{for } -\infty < x < \infty \tag{30}$$

Hence, from Eq. (20), one has

$$p(x,0;t) = \frac{\rho U^2}{2L^2}\, e^{i\omega t} \sum_{m=1}^{\infty} (\mathfrak{B}_m e^{im\pi x/L} - \mathfrak{B}_{-m} e^{-im\pi x/L}) a_m \tag{31}$$

where

$$\mathfrak{B}_m = \frac{(k + m\pi)^2}{\gamma_m}\, \mathfrak{M}(\kappa_m) \qquad \mathfrak{B}_{-m} = \frac{(k - m\pi)^2}{\gamma_{-m}}\, \mathfrak{M}(\kappa_{-m}) \tag{32}$$

The second alternative assumes that the change of aerodynamic pressure due to boundary layer on a finite panel is the same as that on an infinite wall with the same wave form repeated periodically. Thus, if $z_0(x,t)$ is given by (29), one assumes that

$$\text{pressure on wall} = p_0 + \Delta p \tag{33}$$

where p_0 is the wall pressure corresponding to (29) in a potential flow without a boundary layer, and Δp is the difference of p from Eqs. (20) and (22). The function $p_0(x,t)$ is known (see Miles[29]). Simplifications are discussed by Luke[30] and Lock and Fung.[14] The function Δp is

$$\Delta p = \frac{\rho U^2}{2L^2}\, e^{i\omega t} \sum_{m=1}^{\infty} \left\{ \left[\mathfrak{B}_m - \frac{(k + m\pi)^2}{\gamma_m} \right] e^{im\pi x/L} - \left[\mathfrak{B}_{-m} - \frac{(k - m\pi)^2}{\gamma_{-m}} \right] e^{-im\pi x/L} \right\} a_m \tag{34}$$

The first alternative is used in the flutter calculations to be discussed in the next section, in which the analysis is extended to circular cylinders.

It is hoped that such a simple analysis can be supplemented by a comparison of the final results with those obtained in more exact theories. A theoretical-empirical scheme of fixing δ, M_δ, T_δ might be evolved which could be sufficiently accurate for practical purposes. This, however, has not been done yet. Two improved theories have been published so far. One, due to Miles,[31] considers the boundary layer as an inviscid, parallel shear flow over an infinite, plane panel. The other, due to McClure,[32] treats the full problem in the Heisenberg, Tollmien, Lin, Lees, Lighthill tradition, extending the boundary layer problem to oscillating walls. Both Miles and McClure applied their theories to panel flutter. Miles[31] showed that, for a circular cylinder, the stability boundary of short-wavelength traveling waves does not change much on account of the shear layer, but the rate of divergence in the unstable regime may be reduced by an order of magnitude. McClure's solution of the transonic flutter of a flat plate is truly remarkable. In application to the Lock-Fung experiment, McClure obtained the stability boundary as shown in Fig. 3, which is rather close to the experimental value. However, it must be remembered that McClure ignores the leading edge effect in the manner of Eq. (31). How the leading edge effect would influence McClure's stability boundary is yet unknown.

Miles and McClure's analyses are, of course, much more complicated than what was presented here. There are still weakness and difficulties that make an extension of their solutions to an arbitrary wall oscillation impossible. Fortunately, the flexible wall problem has attracted much attention among aero- and hydrodynamicists recently, owing to a great debate about the possibility of reducing the drag of a body in a flow by elastic walls. The names of Kramer, Benjamin, Landahl, Becker, and Laufer are becoming well known in this newly discovered field. It is expected that the matter will be settled before long and with it the aerodynamic aspects on panel flutter, but the matter will not be discussed further here.

6. Theoretical Calculations for Cylindrical Shells

Krumhaar[21] has presented a rigorous analysis of the symmetric flutter of a circular cylindrical shell in a supersonic flow parallel to the cylinder axis, under the assumptions of linear piston theory for aerodynamic pressure and Timoshenko's equations for elastic cylinders. The eigenvalues were found rigorously, thus removing any shadow of doubt caused by the Galerkin approach, which incidentally, was confirmed. (The number of terms required, however, may be quite large.) Krumhaar's results show that the aerodynamic and material damping has tremendous stabilizing influence on the flutter of cylindrical shells in the *symmetric* mode, as was pointed out earlier by Voss.[18]

Krumhaar's analysis yields a set of critical thickness vs Mach number curves as shown in Fig. 15. Here g is the so-called structural damping coefficient. The value of g of the Ames model, estimated from Watts' measurements,[23] is 0.0025. (Such small values of g are pertinent to monocoque missile shells. Commonly used values of damping in ordinary airplane structures for wing flutter analysis are an order of magnitude larger. On the other hand, the internal friction of copper or aluminum at room temperature would be one or two orders of magnitude smaller.) The absence of symmetric flutter from the wind tunnel tests probably is due to material damping.

In Fig. 15, several curves obtained by Anderson[34] also have been included, showing the influence of boundary layer flow on symmetric flutter. The assumptions of Anderson's calculations will be discussed later, but note here that the influence of the boundary layer on the critical flutter condition is not equivalent to material damping.

Voss[18] also showed that material damping has a much smaller influence on flutter of cylindrical shells in the "scallop" modes, with many longitudinal nodal lines uniformly spaced along the circumference of a circular cross section. In contrast, the boundary layer flow has profound influences on the scallop modes of flutter. The last conclusion was reached in Anderson's analysis[34] of the boundary layer in a manner analogous to that presented in the preceding section.

The idealized problem is posed as follows. A uniform parallel subsonic layer of constant thickness exists between a uniform supersonic flow with velocities in the x direction and a circular cylinder whose centerline coincides with the x axis. A small disturbance is induced by an oscillation of the wall of the cylinder. It is desired to find the aerodynamic pressure induced on the wall. The results then are used to analyze the flutter of a thin shell of finite length.

Notations and linearized potential equations and boundary conditions are the same as those used in the flat plate case. In cylindrical polar coordinates, consider an upwash distribution on the wall, $r = R$, of the form

$$w(x,R,\theta,t) = w_0 \sin(m\pi x/L) \cos n\theta \; e^{i\omega t} \qquad (35)$$

The boundary conditions for the velocity potentials ϕ and ϕ_δ are the same as Eqs. (3–7), except that $\partial/\partial y$ is replaced by $\partial/\partial r$. For simplicity, the pressure on the interface $r = R + \delta + z_1(x, \theta, t)$, induced by the oscillation of the interface, is assumed to be given by the two-dimensional linear piston theory. (This assumption introduces errors when the circumferential wave number is large and should be removed in future work, see Krumhaar.[35]) The solution yields the pressure on the cylinder wall corresponding to (35):

$$p(x,R,\theta,t) = (w_0 \rho_\delta \beta_\delta U_\delta / 2M_\delta) e^{i\omega t} \cos n\theta \times \{[\alpha_1]_{mn} e^{-i(m\pi x/L)} - [\alpha_2]_{mn} e^{i(m\pi x/L)}\} \qquad (36)$$

The expression for $[\alpha_1]_{mn}$ and $[\alpha_2]_{mn}$ involves Bessel functions in a rather complex manner. By superposition, the pressure due to an arbitrary oscillation of the cylinder wall can be obtained. Then the Lagrangian equations of motion can be set up, and the critical flutter condition can be derived by Galerkin's method. The details of this lengthy calculation were carried out by Anderson and are presented in Ref. 34.

The model of the boundary layer again is defective, and it is necessary to develop a scheme for replacing a physical boundary layer with an appropriate idealized boundary layer. A tentative scheme is as follows. A typical cross section of the cylinder is chosen at which the boundary layer profile is known. Leave M_δ arbitrary, assume T_δ to be the adiabatic wall temperature, and determine the other parameters U_δ, ρ_δ, and δ according to the assumed M_δ and T_δ and the usual requirements that $p_\delta = p$ in the undisturbed flow, that the loss of volume flow through the idealized boundary layer is the same as the loss in volume flow through the actual boundary layer, and that the boundary layer is turbulent. With this scheme, some detailed comparison of the aerodynamic pressure computed by the foregoing model and that given by linear piston theory is obtained. Attenuation in forces and change in phase shift between force and displacement are particularly evident when n is large (i.e., for scallop modes with many longitudinal nodes). The frequency dependence of these changes is rather complicated.[34]

When the theory is applied to the scallop flutter of a thin-walled circular cylinder of finite length L, with Donnell's equation describing the elastic behavior of the shell and assuming freely supported edge conditions, the results shown in Fig. 16 are obtained, which refer to a copper cylinder with $L/R = 2$ and show the effect of boundary layer thickness on the flutter boundary, the ordinate being proportional to the thickness ratio required to prevent flutter. The very significant effect of boundary layer is seen. Figure 17 shows the influence of the structural damping g on the scallop mode of flutter. The g factor was introduced by multiplying all

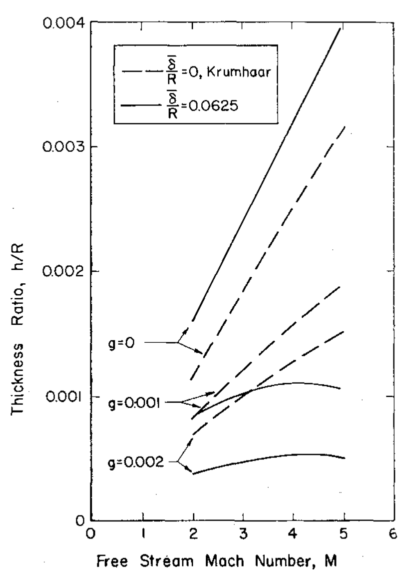

Fig. 15 Effect of material damping and boundary layer on shell thickness required to prevent symmetric flutter. Copper cylinder at altitude 50,000 ft; $R/L = 0.5$, $M_\delta = 0.5$

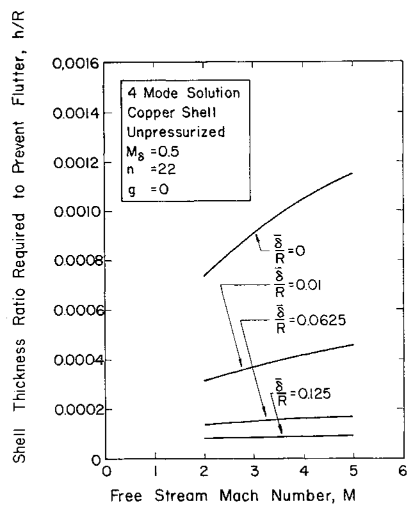

Fig. 16 Effect of boundary layer thickness on the shell thickness required to prevent flutter in scallop mode; copper cylinder at 50,000-ft altitude

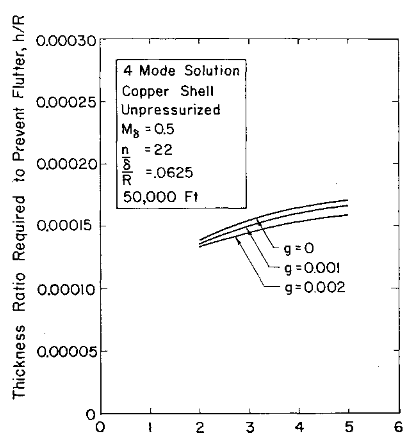

Fig. 17 Influence of structural damping on the scallop mode of flutter of a copper cylinder

elastic restoring force terms by a factor $(1 + ig)$. It is seen that the influence of g is small. Other results concerning the effect of varying n, M_δ, and p_m are given in Ref. 34.

The symmetric mode of flutter, $n = 0$, is a class in itself. Some results are indicated in Fig. 15.

7. Role of Structural Details

The importance of aerodynamic details in the flutter prediction has been discussed in the foregoing. It requires, perhaps, no great persuasion to stress also the importance of structural details. Although in exploratory papers simplified structures suffice, in engineering designs it is desirable to be more careful. Voss[18] has emphasized the importance of membrane stress and inertial forces in the circumferential and longitudinal directions in problems of cylindrical shells. Many papers on vibrations of cylinders have demonstrated the large differences in the frequency spectra when the edge conditions are changed from freely supported conditions to clamped edges. Recent examination of the related problem of buckling of thin shells again calls attention to the great importance in satisfying the realistic edge conditions.

A spacecraft may use nonconventional structures that have no counterpart in conventional theories of plates and shells. An example of corrugation-stiffened plates is discussed by Fung.[36] For such a plate the twisting moments in two orthogonal directions are unequal, thus violating a basic relation in the conventional plate theory.

8. Conclusions

The sizable effort in recent years concerning panel flutter has produced a number of theories with varied assumptions. The purpose of this article is to survey these assumptions in the light of experimental evidence. The reader probably is struck by the meagerness of definitive experimental data. This attests to the difficulty in performing panel flutter tests. Small details of the model construction, edge conditions, and boundary layer are all important. Therefore, as the first conclusion, the author would state that, in the future, more decisive, well-controlled experiments in panel flutter should be performed to supply basic data against which theoretical analyses can be checked.

The author believes that an objective of further panel flutter research is to evolve with a computing program of known accuracy which is sufficiently flexible to meet the needs of varied designs of flight structures. At the present time, crude estimates of panel flutter boundary, within an order of magnitude, already are available, but these estimates cannot meet the needs of the industry. The hope of providing a set of simple empirical curves, based either on experiments or on calculations, might well be dispelled.

To help formulate a computing program, the following conclusions and recommendations may be useful:

1) The viscous boundary layer flow can have important influence on flutter boundaries. For a flat plate, the boundary layer becomes important at lower supersonic Mach numbers ($M < 1.4$). For cylindrical shells, the boundary layer has a large effect with respect to "scallop" modes of flutter (with a number of axial nodal lines along the circumference), even at high Mach numbers.

2) The influence of boundary layer is generally stabilizing. In other words, ignorance of boundary layer would lead to conservative design criteria. However, the interaction between boundary layer damping and structural damping is somewhat unpredictable. For symmetric flutter of cylindrical shells, for which structural damping has large influence, whether ignoring the boundary layer is conservative or not is uncertain.

3) The linear piston theory, the static approximation, or the quasi-steady approximation of aerodynamic pressure should not be used for scallop modes of flutter of cylindrical shells, if the number of circumferential nodes is large (n of order 10).

4) The edge conditions of the panel have profound influence on the stability. For complicated structures such as corrugation-stiffened panels, small details of edge conditions may influence significantly the frequency spectrum of the panel and the panel flutter boundary.

5) A buckled plate or a buckled shell may or may not be more stable than an unbuckled one, depending on the edge conditions, the direction of the compressive loads, and the amplitude of the buckle. Large buckles caused by compression perpendicular to the direction of flow, with nodal lines parallel to the flow, tend to stabilize the panel. Buckling caused by compressive loads parallel to the direction of flow seems to be strongly destabilizing and may induce large amplitude flutter.

6) A rewarding topic for future research is to study panel flutter as a problem in nonlinear mechanics. The author has emphasized the difference in the practical definition of flutter in experiments and that in a linearized theory. So far, the identification of experiments with theory is intuitive. It is evident that a realistic theory is desired.

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